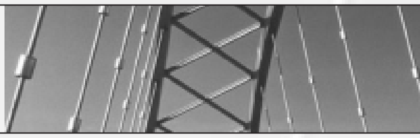


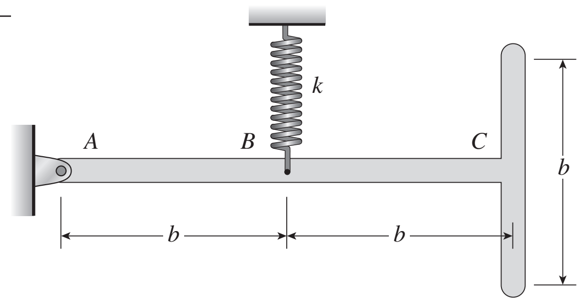
2

Axially Loaded Numbers

Changes in Lengths of Axially Loaded Members

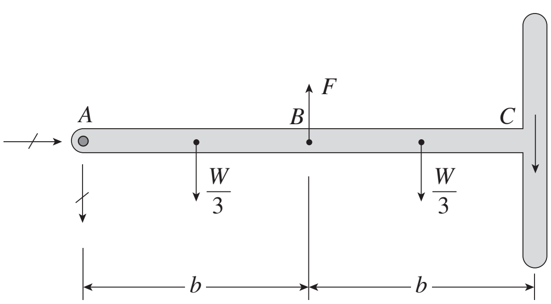
Problem 2.2-1 The T-shaped arm ABC shown in the figure lies in a vertical plane and pivots about a horizontal pin at A. The arm has constant cross-sectional area and total weight W. A vertical spring of stiffness k supports the arm at point B.

Obtain a formula for the elongation δ of the spring due to the weight of the arm.



Solution 2.2-1 **T-shaped arm**

FREE-BODY DIAGRAM OF ARM



F = tensile force in the spring

$\Sigma M_A = 0$

$$F(b) - \frac{W}{3}\left(\frac{b}{2}\right) - \frac{W}{3}\left(\frac{3b}{2}\right) - \frac{W}{3}(2b) = 0$$

$$F = \frac{4W}{3}$$

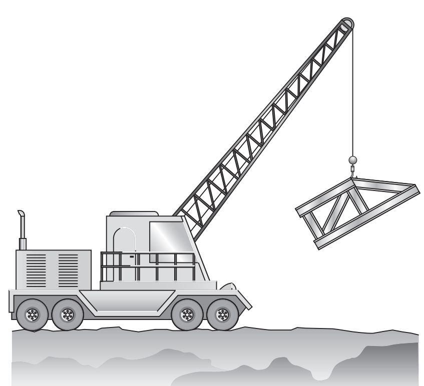
δ = elongation of the spring

$$\delta = \frac{F}{k} = \frac{4W}{3k} \longleftarrow$$

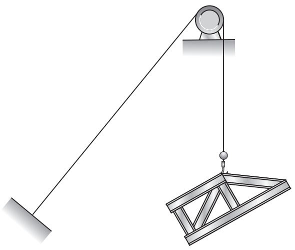
Problem 2.2-2 A steel cable with nominal diameter 25 mm (see Table 2-1) is used in a construction yard to lift a bridge section weighing 38 kN, as shown in the figure. The cable has an effective modulus of elasticity $E = 140$ GPa.

(a) If the cable is 14 m long, how much will it stretch when the load is picked up?

(b) If the cable is rated for a maximum load of 70 kN, what is the factor of safety with respect to failure of the cable?



Solution 2.2-2 **Bridge section lifted by a cable**



$A = 304\ \text{mm}^2$ (from Table 2-1)

$W = 38$ kN

$E = 140$ GPa

$L = 14$ m

(a) Stretch of cable

$$\delta = \frac{WL}{EA} = \frac{(38\ \text{kN})(14\ \text{m})}{(140\ \text{GPa})(304\ \text{mm}^2)}$$

$$= 12.5\ \text{mm} \longleftarrow$$

(b) Factor of safety

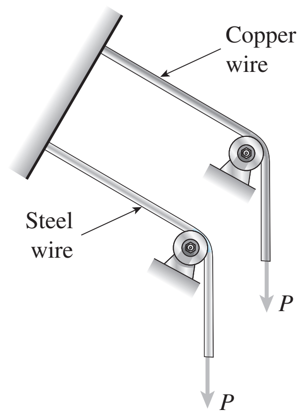
$P_{ULT} = 406$ kN (from Table 2-1)

$P_{\max} = 70$ kN

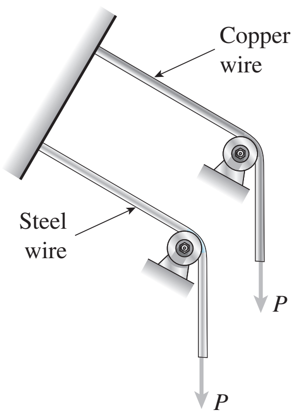
$$n = \frac{P_{ULT}}{P_{\max}} = \frac{406\ \text{kN}}{70\ \text{kN}} = 5.8 \longleftarrow$$

Problem 2.2-3 A steel wire and a copper wire have equal lengths and support equal loads P (see figure). The moduli of elasticity for the steel and copper are $E_s = 30{,}000$ ksi and $E_c = 18{,}000$ ksi, respectively.

(a) If the wires have the same diameters, what is the ratio of the elongation of the copper wire to the elongation of the steel wire?
(b) If the wires stretch the same amount, what is the ratio of the diameter of the copper wire to the diameter of the steel wire?



Solution 2.2-3 **Steel wire and copper wire**



Equal lengths and equal loads

Steel: $E_s = 30{,}000$ ksi

Copper: $E_c = 18{,}000$ ksi

(a) Ratio of elongations (equal diameters)

$$\delta_c = \frac{PL}{E_c A} \qquad \delta_s = \frac{PL}{E_s A}$$

$$\frac{\delta_c}{\delta_s} = \frac{E_s}{E_c} = \frac{30}{18} = 1.67 \longleftarrow$$

(b) Ratio of diameters (equal elongations)

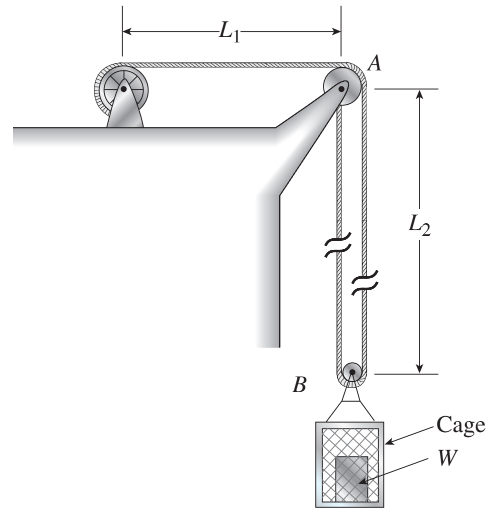
$$\delta_c = \delta_s \qquad \frac{PL}{E_c A_c} = \frac{PL}{E_s A_s} \quad \text{or} \quad E_c A_c = E_s A_s$$

$$E_c\left(\frac{\pi}{4}\right)d_c^2 = E_s\left(\frac{\pi}{4}\right)d_s^2$$

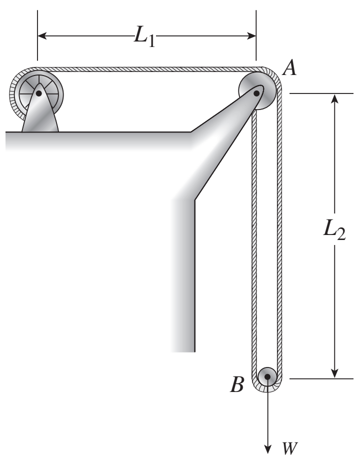
$$\frac{d_c^2}{d_s^2} = \frac{E_s}{E_c} \quad \frac{d_c}{d_s} = \sqrt{\frac{E_s}{E_c}} = \sqrt{\frac{30}{18}} = 1.29 \longleftarrow$$

Problem 2.2-4 By what distance h does the cage shown in the figure move downward when the weight W is placed inside it?

Consider only the effects of the stretching of the cable, which has axial rigidity $EA = 10{,}700$ kN. The pulley at A has diameter $d_A = 300$ mm and the pulley at B has diameter $d_B = 150$ mm. Also, the distance $L_1 = 4.6$ m, the distance $L_2 = 10.5$ m, and the weight $W = 22$ kN. (*Note:* When calculating the length of the cable, include the parts of the cable that go around the pulleys at A and B.)



Solution 2.2-4 Cage supported by a cable



$d_A = 300$ mm

$d_B = 150$ mm

$L_1 = 4.6$ m

$L_2 = 10.5$ m

$EA = 10{,}700$ kN

$W = 22$ kN

Tensile force in cable

$$T = \frac{W}{2} = 11 \text{ kN}$$

Length of cable

$$L = L_1 + 2L_2 + \frac{1}{4}(\pi d_A) + \frac{1}{2}(\pi d_B)$$

$$= 4{,}600 \text{ mm} + 21{,}000 \text{ mm} + 236 \text{ mm} + 236 \text{ mm}$$

$$= 26{,}072 \text{ mm}$$

Elongation of cable

$$\delta = \frac{TL}{EA} = \frac{(11 \text{ kN})(26{,}072 \text{ mm})}{(10{,}700 \text{ kN})} = 26.8 \text{ mm}$$

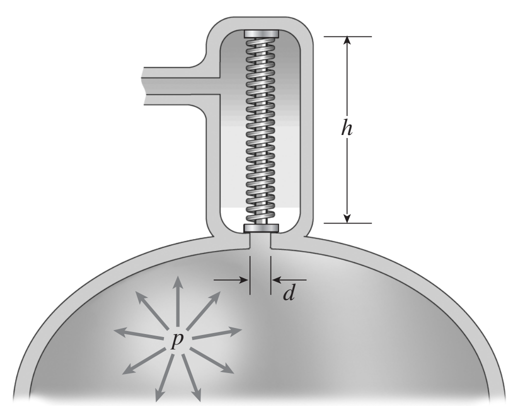
Lowering of the cage

h = distance the cage moves downward

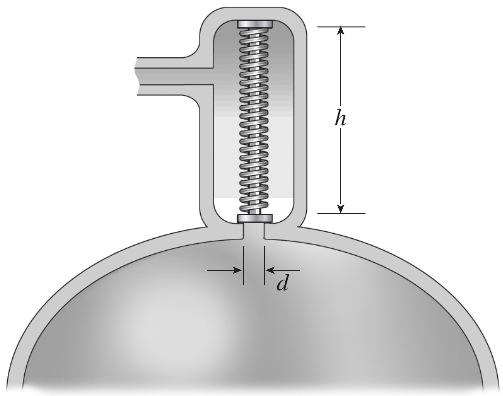
$$h = \frac{1}{2}\delta = 13.4 \text{ mm} \quad \longleftarrow$$

Problem 2.2-5 A safety valve on the top of a tank containing steam under pressure p has a discharge hole of diameter d (see figure). The valve is designed to release the steam when the pressure reaches the value p_{max}.

If the natural length of the spring is L and its stiffness is k, what should be the dimension h of the valve? (Express your result as a formula for h.)



Solution 2.2-5 Safety valve



h = height of valve (compressed length of the spring)

d = diameter of discharge hole

P = pressure in tank

p_{max} = pressure when valve opens

L = natural length of spring ($L > h$)

k = stiffness of spring

FORCE IN COMPRESSED SPRING

$F = k(L - h)$ (From Eq. 2-1a)

PRESSURE FORCE ON SPRING

$$P = p_{max}\left(\frac{\pi d^2}{4}\right)$$

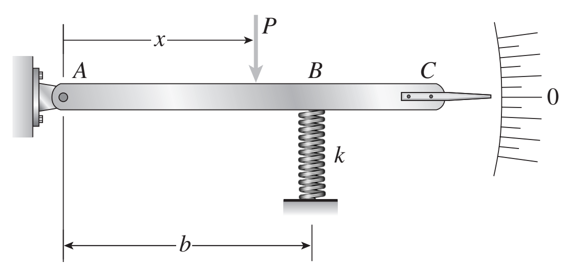
EQUATE FORCES AND SOLVE FOR h:

$$F = P \quad k(L - h) = \frac{\pi p_{max}\, d^2}{4}$$

$$h = L - \frac{\pi p_{max}\, d^2}{4k} \quad \longleftarrow$$

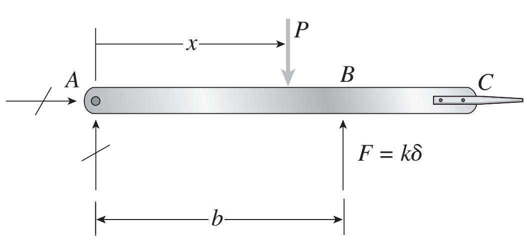
Problem 2.2-6 The device shown in the figure consists of a pointer ABC supported by a spring of stiffness k = 800 N/m. The spring is positioned at distance b = 150 mm from the pinned end A of the pointer. The device is adjusted so that when there is no load P, the pointer reads zero on the angular scale.

If the load P = 8 N, at what distance x should the load be placed so that the pointer will read 3° on the scale?



Solution 2.2-6 Pointer supported by a spring

FREE-BODY DIAGRAM OF POINTER



P = 8 N

k = 800 N/m

b = 150 mm

δ = displacement of spring

F = force in spring

$= k\delta$

$$\Sigma M_A = 0$$

$$-Px + (k\delta)b = 0 \quad \text{or} \quad \delta = \frac{Px}{kb}$$

Let α = angle of rotation of pointer

$$\tan\alpha = \frac{\delta}{b} = \frac{Px}{kb^2} \qquad x = \frac{kb^2}{P}\tan\alpha \quad \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

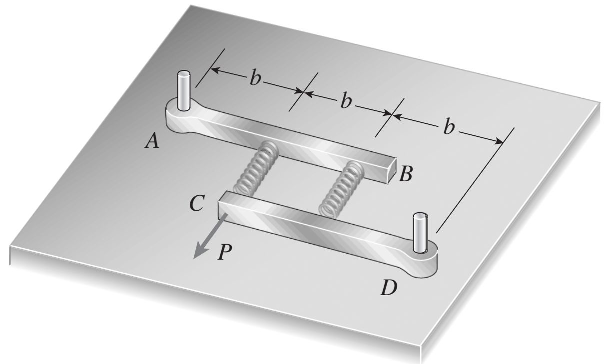
$\alpha = 3°$

$$x = \frac{(800\text{ N/m})(150\text{ mm})^2}{8\text{ N}}\tan 3°$$

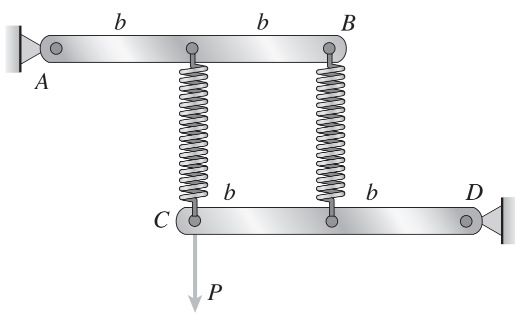
$$= 118\text{ mm} \quad \longleftarrow$$

Problem 2.2-7 Two rigid bars, *AB* and *CD*, rest on a smooth horizontal surface (see figure). Bar *AB* is pivoted end *A* and bar *CD* is pivoted at end *D*. The bars are connected to each other by two linearly elastic springs of stiffness *k*. Before the load *P* is applied, the lengths of the springs are such that the bars are parallel and the springs are without stress.

Derive a formula for the displacement δ_C at point *C* when the load *P* is acting. (Assume that the bars rotate through very small angles under the action of the load *P*.)



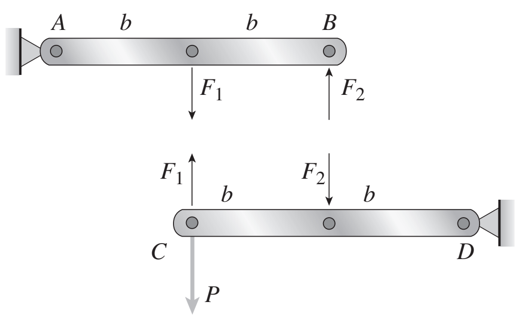
Solution 2.2-7 **Two bars connected by springs**



k = stiffness of springs

δ_C = displacement at point C due to load P

FREE-BODY DIAGRAMS



F_1 = tensile force in first spring

F_2 = compressive force in second spring

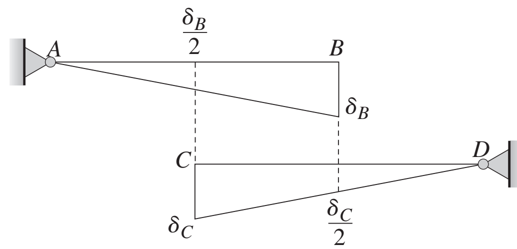
EQUILIBRIUM

$\Sigma M_A = 0 \qquad -bF_1 + 2bF_2 = 0 \quad F_1 = 2F_2$

$\Sigma M_D = 0 \quad 2bP - 2bF_1 + bF_2 = 0 \quad F_2 = 2F_1 - 2P$

Solving, $F_1 = \dfrac{4P}{3} \quad F_2 = \dfrac{2P}{3}$

DISPLACEMENT DIAGRAMS



δ_B = displacement of point B

δ_C = displacement of point C

Δ_1 = elongation of first spring

$= \delta_C - \dfrac{\delta_B}{2}$

Δ_2 = shortening of second spring

$= \delta_B - \dfrac{\delta_C}{2}$

Also, $\quad \Delta_1 = \dfrac{F_1}{k} = \dfrac{4P}{3k}; \qquad \Delta_2 = \dfrac{F_2}{k} = \dfrac{2P}{3k}$

SOLVE THE EQUATIONS:

$$\Delta_1 = \Delta_1 \quad \delta_C - \frac{\delta_B}{2} = \frac{4P}{3k}$$

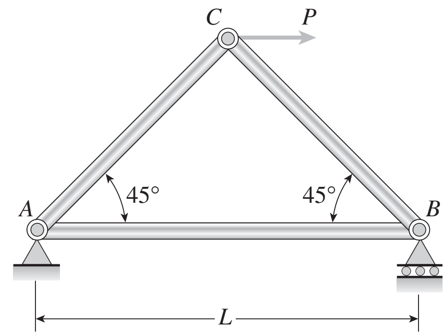
$$\Delta_2 = \Delta_2 \quad \delta_B - \frac{\delta_C}{2} = \frac{2P}{3k}$$

Eliminate δ_B and obtain δ_C:

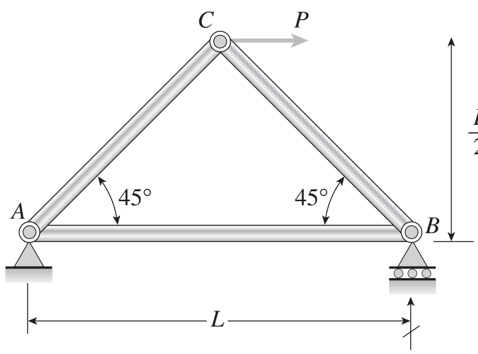
$$\delta_C = \frac{20P}{9k} \quad \longleftarrow$$

Problem 2.2-8 The three-bar truss *ABC* shown in the figure has a span $L = 3$ m and is constructed of steel pipes having cross-sectional area $A = 3900 \text{ mm}^2$ and modulus of elasticity $E = 200$ GPa. A load P acts horizontally to the right at joint C.

(a) If $P = 650$ kN, what is the horizontal displacement of joint B?
(b) What is the maximum permissible load P_{max} if the displacement of joint B is limited to 1.5 mm?



Solution 2.2-8 **Truss with horizontal load**



$L = 3$ m

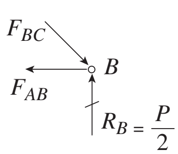
$A = 3900 \text{ mm}^2$

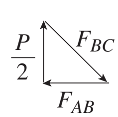
$E = 200$ GPa

$\Sigma M_A = 0 \quad \text{gives} \quad R_B = \dfrac{P}{2}$

FREE-BODY DIAGRAM OF JOINT B

Force triangle:





From force triangle,

$$F_{AB} = \frac{P}{2} \text{ (tension)}$$

(a) HORIZONTAL DISPLACEMENT δ_B

$P = 650$ kN

$$\delta_B = \frac{F_{AB}\,L_{AB}}{EA} = \frac{PL}{2EA}$$

$$= \frac{(650 \text{ kN})(3 \text{ m})}{2(200 \text{ GPa})(3900 \text{ mm}^2)}$$

$$= 1.25 \text{ mm} \longleftarrow$$

(b) MAXIMUM LOAD P_{max}

$\delta_{max} = 1.5$ mm

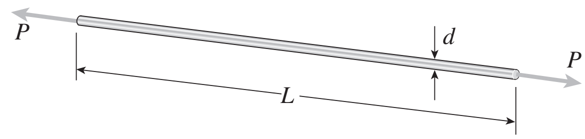
$$\frac{P_{max}}{\delta_{max}} = \frac{P}{\delta} \qquad P_{max} = P\left(\frac{\delta_{max}}{\delta}\right)$$

$$P_{max} = (650 \text{ kN})\left(\frac{1.5 \text{ mm}}{1.25 \text{ mm}}\right)$$

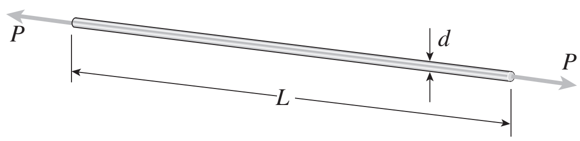
$$= 780 \text{ kN} \longleftarrow$$

Problem 2.2-9 An aluminum wire having a diameter $d = 2$ mm and length $L = 3.8$ m is subjected to a tensile load P (see figure). The aluminum has modulus of elasticity $E = 75$ GPa.

If the maximum permissible elongation of the wire is 3.0 mm and the allowable stress in tension is 60 MPa, what is the allowable load P_{max}?



Solution 2.2-9 Aluminum wire in tension



$d = 2$ mm

$L = 3.8$ m

$E = 75$ GPa

$$A = \frac{\pi d^2}{4} = 3.142 \text{ mm}^2$$

MAXIMUM LOAD BASED UPON ELONGATION

$$\delta_{max} = 3.0 \text{ mm} \qquad \delta = \frac{PL}{EA}$$

$$P_{max} = \frac{\text{EA}}{\text{L}}\delta_{max}$$

$$= \frac{(75 \text{ GPa})(3.142 \text{ mm}^2)}{3.8 \text{ m}}(3.0 \text{ mm})$$

$$= 186 \text{ N}$$

MAXIMUM LOAD BASED UPON STRESS

$$\sigma_{allow} = 60 \text{ MPa} \qquad \sigma = \frac{P}{A}$$

$$P_{max} = A\sigma_{allow} = (3.142 \text{ mm}^2)(60 \text{ MPa})$$

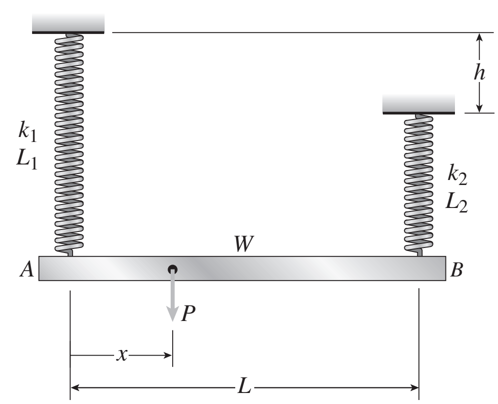
$$= 189 \text{ N}$$

ALLOWABLE LOAD

Elongation governs. $P_{max} = 186 \text{ N} \longleftarrow$

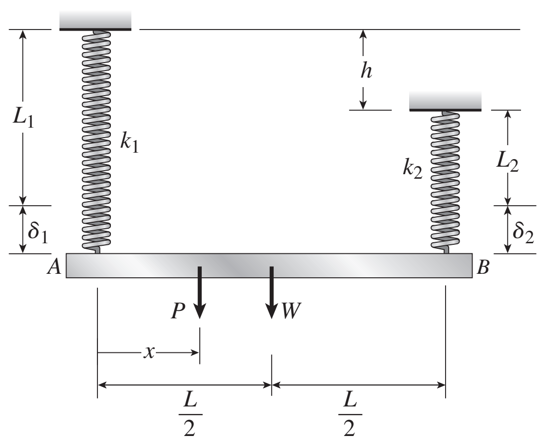
Problem 2.2-10 A uniform bar AB of weight $W = 25$ N is supported by two springs, as shown in the figure. The spring on the left has stiffness $k_1 = 300$ N/m and natural length $L_1 = 250$ mm. The corresponding quantities for the spring on the right are $k_2 = 400$ N/m and $L_2 = 200$ mm. The distance between the springs is $L = 350$ mm, and the spring on the right is suspended from a support that is distance $h = 80$ mm below the point of support for the spring on the left.

At what distance x from the left-hand spring should a load $P = 18$ N be placed in order to bring the bar to a horizontal position?



Solution 2.2-10 Bar supported by two springs

Reference line



$W = 25$ N

$k_1 = 300$ N/m

$k_2 = 400$ N/m

$L = 350$ mm

$h = 80$ mm

$P = 18$ N

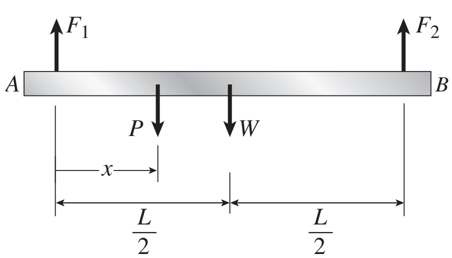
NATURAL LENGTHS OF SPRINGS

$L_1 = 250$ mm $\qquad L_2 = 200$ mm

OBJECTIVE

Find distance x for bar AB to be horizontal.

FREE-BODY DIAGRAM OF BAR AB



$\Sigma M_A = 0$ ↶+ ↷

$$F_2 L - P_X - \frac{WL}{2} = 0 \qquad \text{(Eq. 1)}$$

$$\Sigma F_{\text{vert}} = 0 \quad \uparrow_+ \quad \downarrow^-$$

$$F_1 + F_2 - P - W = 0 \qquad \text{(Eq. 2)}$$

SOLVE EQS. (1) AND (2):

$$F_1 = P\left(1 - \frac{x}{L}\right) + \frac{W}{2} \qquad F_2 = \frac{P_X}{L} + \frac{W}{2}$$

SUBSTITUTE NUMERICAL VALUES:

UNITS: Newtons and meters

$$F_1 = (18)\left(1 - \frac{x}{0.350}\right) + 12.5 = 30.5 - 51.429x$$

$$F_2 = (18)\left(\frac{x}{0.350}\right) + 12.5 = 51.429x + 12.5$$

ELONGATIONS OF THE SPRINGS

$$\delta_1 = \frac{F_1}{k_1} = \frac{F_1}{300} = 0.10167 - 0.17143x$$

$$\delta_2 = \frac{F_2}{k_2} = \frac{F_2}{400} = 0.12857x + 0.031250$$

BAR AB REMAINS HORIZONTAL

Points A and B are the same distance below the reference line (see figure above).

$$\therefore\ L_1 + \delta_1 = h + L_2 + \delta_2$$

or $\quad 0.250 + 0.10167 - 0.17143\,x$
$\quad = 0.080 + 0.200 + 0.12857\,x + 0.031250$

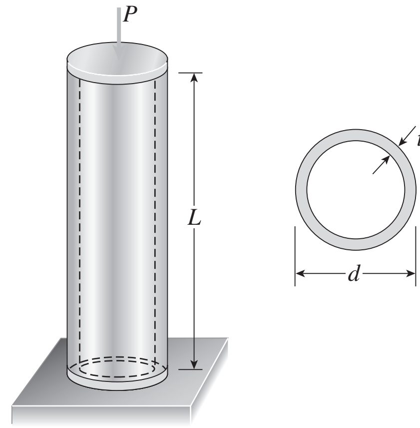
SOLVE FOR x:

$0.300\,x = 0.040420 \qquad x = 0.1347$ m

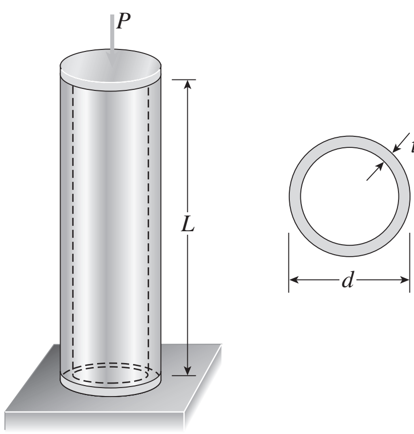
$x = 135$ mm ⟵

Problem 2.2-11 A hollow, circular, steel column ($E = 30{,}000$ ksi) is subjected to a compressive load P, as shown in the figure. The column has length $L = 8.0$ ft and outside diameter $d = 7.5$ in. The load $P = 85$ k.

If the allowable compressive stress is 7000 psi and the allowable shortening of the column is 0.02 in., what is the minimum required wall thickness t_{min}?



Solution 2.2-11 **Column in compression**



$P = 85$ k

$E = 30{,}000$ ksi

$L = 8.0$ ft

$d = 7.5$ in.

$\sigma_{allow} = 7{,}000$ psi

$\delta_{allow} = 0.02$ in.

Required area based upon allowable stress

$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma_{allow}} = \frac{85\text{ k}}{7{,}000\text{ psi}} = 12.14\text{ in.}^2$$

Required area based upon allowable shortening

$$\delta = \frac{PL}{EA} \quad A = \frac{PL}{E\delta_{allow}} = \frac{(85\text{ k})(96\text{ in.})}{(30{,}000\text{ ksi})(0.02\text{ in.})}$$

$$= 13.60\text{ in.}^2$$

Shortening governs

$$A_{min} = 13.60\text{ in.}^2$$

Minimum thickness t_{min}

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2] \quad \text{or} \quad \frac{4A}{\pi} - d^2$$

$$= -(d - 2t)^2$$

$$(d - 2t)^2 = d^2 - \frac{4A}{\pi} \quad \text{or} \quad d - 2t = \sqrt{d^2 - \frac{4A}{\pi}}$$

$$t = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \frac{A}{\pi}} \quad \text{or}$$

$$t_{min} = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \frac{A_{min}}{\pi}}$$

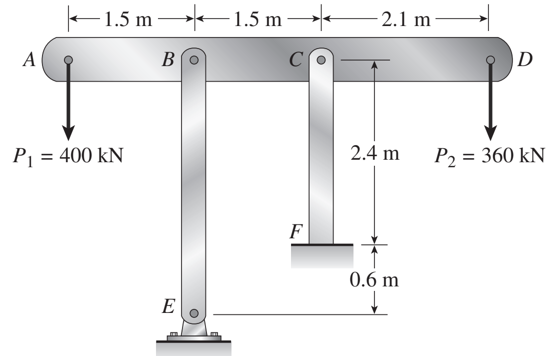
Substitute numerical values

$$t_{min} = \frac{7.5\text{ in.}}{2} - \sqrt{\left(\frac{7.5\text{ in.}}{2}\right)^2 - \frac{13.60\text{ in.}^2}{\pi}}$$

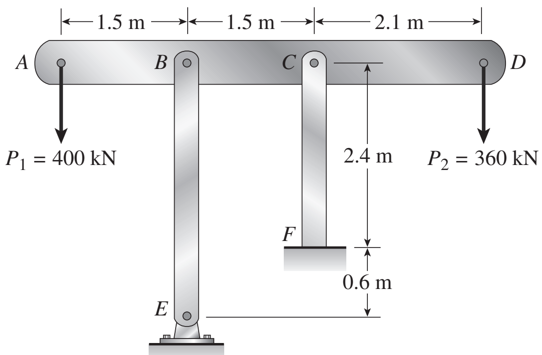
$$t_{min} = 0.63\text{ in.} \longleftarrow$$

Problem 2.2-12 The horizontal rigid beam *ABCD* is supported by vertical bars *BE* and *CF* and is loaded by vertical forces $P_1 = 400$ kN and $P_2 = 360$ kN acting at points *A* and *D*, respectively (see figure). Bars *BE* and *CF* are made of steel ($E = 200$ GPa) and have cross-sectional areas $A_{BE} = 11{,}100\ \text{mm}^2$ and $A_{CF} = 9{,}280\ \text{mm}^2$. The distances between various points on the bars are shown in the figure.

Determine the vertical displacements δ_A and δ_D of points *A* and *D*, respectively.



Solution 2.2-12 **Rigid beam supported by vertical bars**



$A_{BE} = 11{,}100\ \text{mm}^2$

$A_{CF} = 9{,}280\ \text{mm}^2$

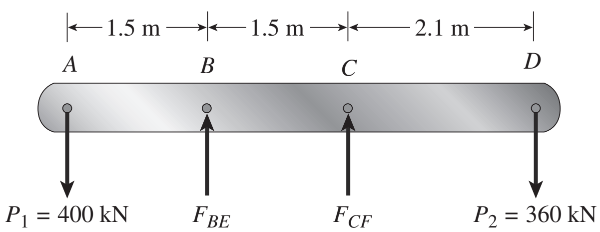
$E = 200$ GPa

$L_{BE} = 3.0$ m

$L_{CF} = 2.4$ m

$P_1 = 400$ kN; $P_2 = 360$ kN

FREE-BODY DIAGRAM OF BAR *ABCD*



$\Sigma M_B = 0$

$(400\text{ kN})(1.5\text{ m}) + F_{CF}(1.5\text{ m}) - (360\text{ kN})(3.6\text{ m}) = 0$

$F_{CF} = 464$ kN

$\Sigma M_C = 0$

$(400\text{ kN})(3.0\text{ m}) - F_{BE}(1.5\text{ m}) - (360\text{ kN})(2.1\text{ m}) = 0$

$F_{BE} = 296$ kN

SHORTENING OF BAR *BE*

$$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA_{BE}} = \frac{(296\text{ kN})(3.0\text{ m})}{(200\text{ GPa})(11{,}100\text{ mm}^2)}$$

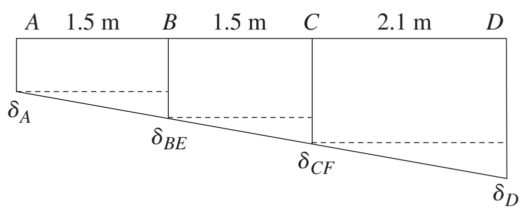
$$= 0.400\text{ mm}$$

SHORTENING OF BAR *CF*

$$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA_{CF}} = \frac{(464\text{ kN})(2.4\text{ m})}{(200\text{ GPa})(9{,}280\text{ mm}^2)}$$

$$= 0.600\text{ mm}$$

DISPLACEMENT DIAGRAM



$\delta_{BE} - \delta_A = \delta_{CF} - \delta_{BE}$ or $\delta_A = 2\delta_{BE} - \delta_{CF}$

$\delta_A = 2(0.400\text{ mm}) - 0.600$ m

$= 0.200$ mm ⟵

(Downward)

$$\delta_D - \delta_{CF} = \frac{2.1}{1.5}(\delta_{CF} - \delta_{BE})$$

or $$\delta_D = \frac{12}{5}\delta_{CF} - \frac{7}{5}\delta_{BE}$$

$$= \frac{12}{5}(0.600\text{ mm}) - \frac{7}{5}(0.400\text{ mm})$$

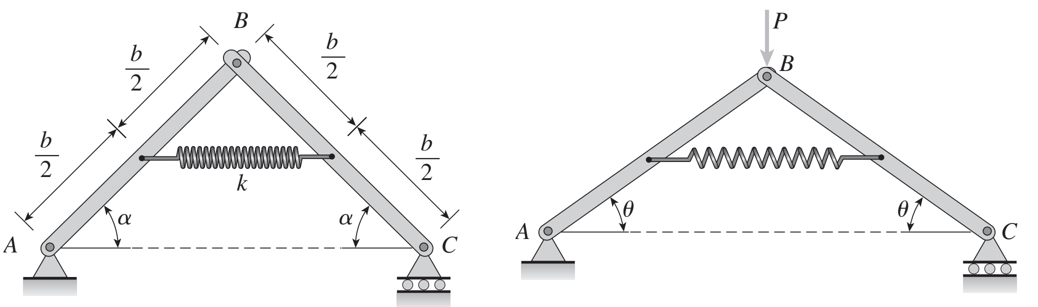
$= 0.880$ mm ⟵

(Downward)

Problem 2.2-13 A framework ABC consists of two rigid bars AB and BC, each having length b (see the first part of the figure). The bars have pin connections at A, B, and C and are joined by a spring of stiffness k. The spring is attached at the midpoints of the bars. The framework has a pin support at A and a roller support at C, and the bars are at an angle α to the hoizontal.

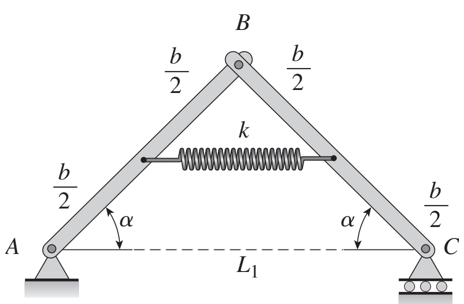
When a vertical load P is applied at joint B (see the second part of the figure) the roller support C moves to the right, the spring is stretched, and the angle of the bars decreases from α to the angle θ.

Determine the angle θ and the increase δ in the distance between points A and C. (Use the following data; $b = 8.0$ in., $k = 16$ lb/in., $\alpha = 45°$, and $P = 10$ lb.)



Probs. 2.1-13 and 2.2-14

Solution 2.2-13 **Framework with rigid bars and a spring**



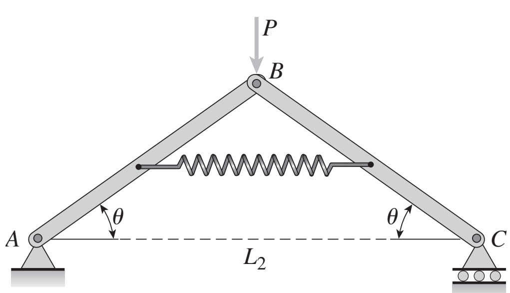
WITH NO LOAD

L_1 = span from A to C

$= 2b \cos \alpha$

S_1 = length of spring

$$= \frac{L_1}{2} = b \cos \alpha$$



WITH LOAD P

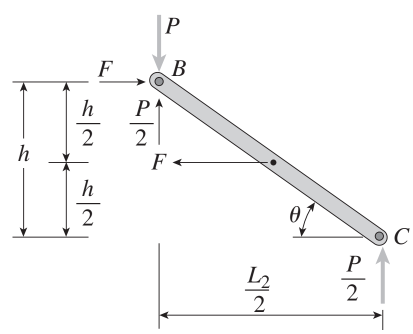
L_2 = span from A to C

$= 2b \cos \theta$

S_2 = length of spring

$$= \frac{L_2}{2} = b \cos \theta$$

FREE-BODY DIAGRAM OF BC



h = height from C to $B = b \sin \theta$

$$\frac{L_2}{2} = b \cos \theta$$

F = force in spring due to load P

$\Sigma M_B = 0$

$$\frac{P}{2}\left(\frac{L_2}{2}\right) - F\left(\frac{h}{2}\right) = 0 \text{ or } P \cos \theta = F \sin \theta \qquad \text{(Eq. 1)}$$

DETERMINE THE ANGLE θ

ΔS = elongation of spring

$= S_2 - S_1 = b(\cos\theta - \cos\alpha)$

For the spring: $F = k(\Delta S)$

$F = bk(\cos\theta - \cos\alpha)$

Substitute F into Eq. (1):

$P\cos\theta = bk(\cos\theta - \cos\alpha)(\sin\theta)$

or $\quad \dfrac{P}{bk}\cot\theta - \cos\theta + \cos\alpha = 0 \longleftarrow \qquad$ (Eq. 2)

This equation must be solved numerically for the angle θ.

DETERMINE THE DISTANCE δ

$\delta = L_2 - L_1 = 2b\cos\theta - 2b\cos\alpha$

$= 2b(\cos\theta - \cos\alpha)$

From Eq. (2): $\cos\alpha = \cos\theta - \dfrac{P\cot\theta}{bk}$

Therefore,

$$\delta = 2b\left(\cos\theta - \cos\theta + \frac{P\cot\theta}{bk}\right)$$

$$= \frac{2P}{b}\cot\theta \longleftarrow \qquad \text{(Eq. 3)}$$

NUMERICAL RESULTS

$b = 8.0$ in. $\quad k = 16$ lb/in. $\quad \alpha = 45°\quad P = 10$ lb

Substitute into Eq. (2):

$0.078125\cot\theta - \cos\theta + 0.707107 = 0 \qquad$ (Eq. 4)

Solve Eq. (4) numerically:

$\theta = 35.1° \longleftarrow$

Substitute into Eq. (3):

$\delta = 1.78$ in. $\longleftarrow$

Problem 2.2-14 Solve the preceding problem for the following data: $b = 200$ mm, $k = 3.2$ kN/m, $\alpha = 45°$, and $P = 50$ N.

Solution 2.2-14 Framework with rigid bars and a spring

See the solution to the preceding problem.

EQ. (2): $\quad \dfrac{P}{bk}\cot\theta - \cos\theta + \cos\alpha = 0$

EQ. (3): $\quad \delta = \dfrac{2P}{k}\cot\theta$

NUMERICAL RESULTS

$b = 200$ mm $\quad k = 3.2$ kN/m $\quad \alpha = 45°\quad P = 50$ N

Substitute into Eq. (2):

$0.078125\cot\theta - \cos\theta + 0.707107 = 0 \qquad$ (Eq. 4)

Solve Eq. (4) numerically:

$\theta = 35.1° \longleftarrow$

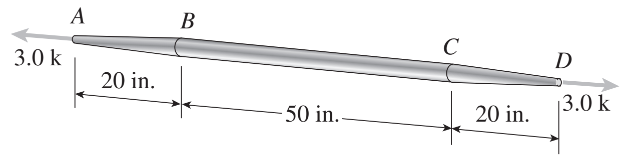
Substitute into Eq. (3):

$\delta = 44.5$ mm $\longleftarrow$

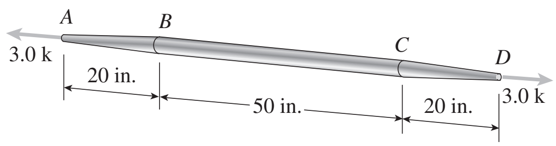
Changes in Lengths Under Nonuniform Conditions

Problem 2.3-1 Calculate the elongation of a copper bar of solid circular cross section with tapered ends when it is stretched by axial loads of magnitude 3.0 k (see figure).

The length of the end segments is 20 in. and the length of the prismatic middle segment is 50 in. Also, the diameters at cross sections A, B, C, and D are 0.5, 1.0, 1.0, and 0.5 in., respectively, and the modulus of elasticity is 18,000 ksi. (*Hint:* Use the result of Example 2-4.)



Solution 2.3-1 Bar with tapered ends



$d_A = d_D = 0.5$ in. $\quad P = 3.0$ k

$d_B = d_C = 1.0$ in. $\quad E = 18{,}000$ ksi

End segment ($L = 20$ in.)

From Example 2-4:

$$\delta = \frac{4PL}{\pi E\, d_A\, d_B}$$

$$\delta_1 = \frac{4(3.0\text{ k})(20\text{ in.})}{\pi(18{,}000\text{ ksi})(0.5\text{ in.})(1.0\text{ in.})} = 0.008488\text{ in.}$$

Middle segment ($L = 50$ in.)

$$\delta_2 = \frac{PL}{EA} = \frac{(3.0\text{ k})(50\text{ in.})}{(18{,}000\text{ ksi})(\frac{\pi}{4})(1.0\text{ in.})^2}$$

$$= 0.01061\text{in.}$$

Elongation of bar

$$\delta = \sum \frac{NL}{EA} = 2\delta_1 + \delta_2$$

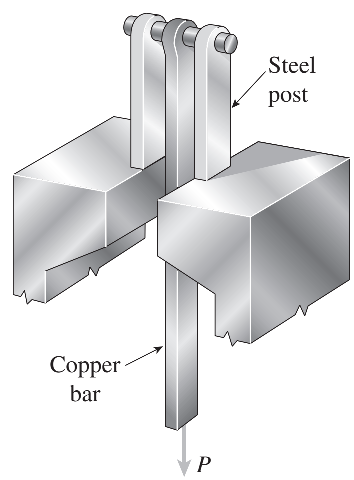
$$= 2(0.008488\text{ in.}) + (0.01061\text{ in.})$$

$$= 0.0276\text{ in.} \longleftarrow$$

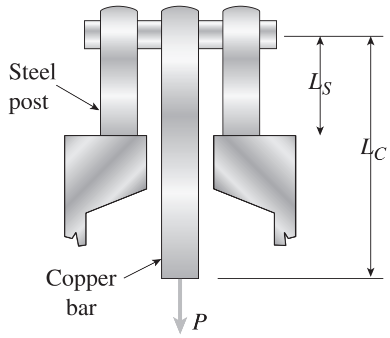
Problem 2.3-2 A long, rectangular copper bar under a tensile load P hangs from a pin that is supported by two steel posts (see figure). The copper bar has a length of 2.0 m, a cross-sectional area of 4800 mm^2, and a modulus of elasticity $E_c = 120$ GPa. Each steel post has a height of 0.5 m, a cross-sectional area of 4500 mm^2, and a modulus of elasticity $E_s = 200$ GPa.

(a) Determine the downward displacement δ of the lower end of the copper bar due to a load $P = 180$ kN.

(b) What is the maximum permissible load P_{max} if the displacement δ is limited to 1.0 mm?



Solution 2.3-2 Copper bar with a tensile load



$L_c = 2.0$ m

$A_c = 4800$ mm²

$E_c = 120$ GPa

$L_s = 0.5$ m

$A_s = 4500$ mm²

$E_s = 200$ GPa

(a) DOWNWARD DISPLACEMENT δ ($P = 180$ kN)

$$\delta_c = \frac{PL_c}{E_c A_c} = \frac{(180 \text{ kN})(2.0 \text{ m})}{(120 \text{ GPa})(4800 \text{ mm}^2)}$$

$$= 0.625 \text{ mm}$$

$$\delta_s = \frac{(P/2)L_s}{E_s A_s} = \frac{(90 \text{ kN})(0.5 \text{ m})}{(200 \text{ GPa})(4500 \text{ mm}^2)}$$

$$= 0.050 \text{ mm}$$

$$\delta = \delta_c + \delta_s = 0.625 \text{ mm} + 0.050 \text{ mm}$$

$$= 0.675 \text{ mm} \longleftarrow$$

(b) MAXIMUM LOAD $P_{\max}$ ($\delta_{\max} = 1.0$ mm)

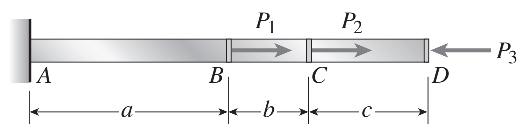
$$\frac{P_{\max}}{P} = \frac{\delta_{\max}}{\delta} \qquad P_{\max} = P\left(\frac{\delta_{\max}}{\delta}\right)$$

$$P_{\max} = (180 \text{ kN})\left(\frac{1.0 \text{ mm}}{0.675 \text{ mm}}\right) = 267 \text{ kN} \longleftarrow$$

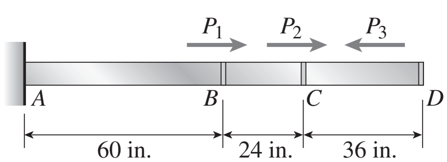
Problem 2.3-3 A steel bar AD (see figure) has a cross-sectional area of 0.40 in.² and is loaded by forces $P_1 = 2700$ lb, $P_2 = 1800$ lb, and $P_3 = 1300$ lb. The lengths of the segments of the bar are $a = 60$ in., $b = 24$ in., and $c = 36$ in.

(a) Assuming that the modulus of elasticity $E = 30 \times 10^6$ psi, calculate the change in length δ of the bar. Does the bar elongate or shorten?

(b) By what amount P should the load P_3 be increased so that the bar does not change in length when the three loads are applied?



Solution 2.3-3 Steel bar loaded by three forces



$A = 0.40$ in.² $\quad P_1 = 2700$ lb $\quad P_2 = 1800$ lb

$P_3 = 1300$ lb $\quad E = 30 \times 10^6$ psi

AXIAL FORCES (t = tension)

$N_{AB} = P_1 + P_2 - P_3 = 3200$ lb

$N_{BC} = P_2 - P_3 = 500$ lb

$N_{CD} = -P_3 = -1300$ lb

(a) CHANGE IN LENGTH

$$\delta = \sum \frac{N_i L_i}{E_i A_i}$$

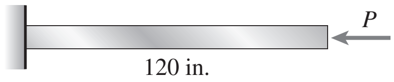
$$= \frac{1}{EA}(N_{AB}L_{AB} + N_{BC}L_{BC} + N_{CD}L_{CD})$$

$$= \frac{1}{(30 \times 10^6 \text{ psi})(0.40 \text{ in.}^2)}[(3200 \text{ lb})(60 \text{ in.})$$

$$+ (500 \text{ lb})(24 \text{ in.}) - (1300 \text{ lb})(36 \text{ in.})]$$

$$= 0.0131 \text{ in. (elongation)} \longleftarrow$$

(b) INCREASE IN P_3 FOR NO CHANGE IN LENGTH



P = increase in force P_3

The force P must produce a shortening equal to 0.0131 in. in order to have no change in length.

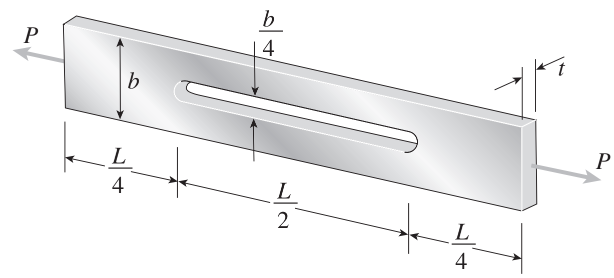
$$\therefore\ 0.0131 \text{ in.} = \delta = \frac{PL}{EA}$$

$$= \frac{P(120 \text{ in.})}{(30 \times 10^6 \text{ psi})(0.40 \text{ in.}^2)}$$

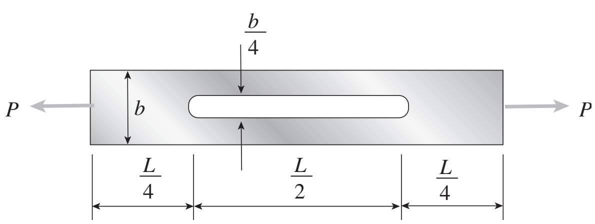
$$P = 1310 \text{ lb} \longleftarrow$$

Problem 2.3-4 A rectangular bar of length L has a slot in the middle half of its length (see figure). The bar has width b, thickness t, and modulus of elasticity E. The slot has width $b/4$.

(a) Obtain a formula for the elongation δ of the bar due to the axial loads P.
(b) Calculate the elongation of the bar if the material is high-strength steel, the axial stress in the middle region is 160 MPa, the length is 750 mm, and the modulus of elasticity is 210 GPa.



Solution 2.3-4 **Bar with a slot**



t = thickness $\quad L$ = length of bar

(a) ELONGATION OF BAR

$$\delta = \sum \frac{N_i L_i}{EA_i} = \frac{P(L/4)}{E(bt)} + \frac{P(L/2)}{E(\frac{3}{4}bt)} + \frac{P(L/4)}{E(bt)}$$

$$= \frac{PL}{Ebt}\left(\frac{1}{4} + \frac{4}{6} + \frac{1}{4}\right) = \frac{7PL}{6Ebt} \longleftarrow$$

STRESS IN MIDDLE REGION

$$\sigma = \frac{P}{A} = \frac{P}{(\frac{3}{4}bt)} = \frac{4P}{3\ bt} \quad \text{or} \quad \frac{P}{bt} = \frac{3\sigma}{4}$$

Substitute into the equation for δ:

$$\delta = \frac{7PL}{6Ebt} = \frac{7L}{6E}\left(\frac{P}{bt}\right) = \frac{7L}{6E}\left(\frac{3\sigma}{4}\right)$$

$$= \frac{7\sigma L}{8E}$$

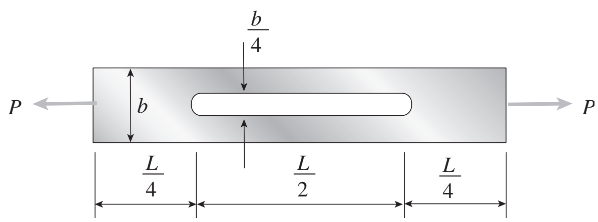
(b) SUBSTITUTE NUMERICAL VALUES:

$\sigma = 160$ MPa $\quad L = 750$ mm $\quad E = 210$ GPa

$$\delta = \frac{7(160 \text{ MPa})(750 \text{ mm})}{8\ (210 \text{ GPa})} = 0.500 \text{ mm} \longleftarrow$$

Problem 2.3-5 Solve the preceding problem if the axial stress in the middle region is 24,000 psi, the length is 30 in., and the modulus of elasticity is 30×10^6 psi.

Solution 2.3-5 Bar with a slot



t = thickness L = length of bar

(a) ELONGATION OF BAR

$$\delta = \sum \frac{N_i L_i}{EA_i} = \frac{P(L/4)}{E(bt)} + \frac{P(L/2)}{E\left(\frac{3}{4}bt\right)} + \frac{P(L/4)}{E(bt)}$$

$$= \frac{PL}{Ebt}\left(\frac{1}{4} + \frac{4}{6} + \frac{1}{4}\right) = \frac{7PL}{6Ebt} \longleftarrow$$

STRESS IN MIDDLE REGION

$$\sigma = \frac{P}{A} = \frac{P}{\left(\frac{3}{4}bt\right)} = \frac{4P}{3bt} \quad \text{or} \quad \frac{P}{bt} = \frac{3\sigma}{4}$$

SUBSTITUTE INTO THE EQUATION FOR δ:

$$\delta = \frac{7PL}{6Ebt} = \frac{7L}{6E}\left(\frac{P}{bt}\right) = \frac{7L}{6E}\left(\frac{3\sigma}{4}\right)$$

$$= \frac{7\sigma L}{8E}$$

(b) SUBSTITUTE NUMERICAL VALUES:

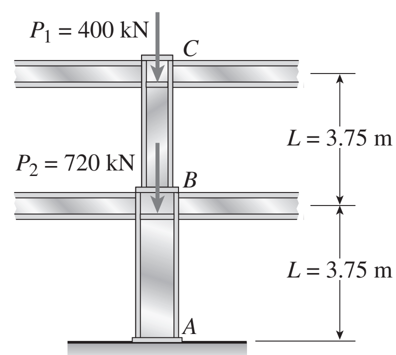
$\sigma = 24{,}000$ psi $L = 30$ in.

$E = 30 \times 10^6$ psi

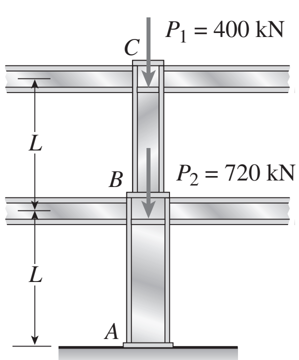
$$\delta = \frac{7(24{,}000 \text{ psi})(30 \text{ in.})}{8(30 \times 10^6 \text{ psi})} = 0.0210 \text{ in.} \longleftarrow$$

Problem 2.3-6 A two-story building has steel columns AB in the first floor and BC in the second floor, as shown in the figure. The roof load P_1 equals 400 kN and the second-floor load P_2 equals 720 kN. Each column has length $L = 3.75$ m. The cross-sectional areas of the first- and second-floor columns are 11,000 mm^2 and 3,900 mm^2, respectively.

(a) Assuming that $E = 206$ GPa, determine the total shortening δ_{AC} of the two columns due to the combined action of the loads P_1 and P_2.
(b) How much additional load P_0 can be placed at the top of the column (point C) if the total shortening δ_{AC} is not to exceed 4.0 mm?



Solution 2.3-6 Steel columns in a building



L = length of each column

$= 3.75$ m

$E = 206$ GPa

$A_{AB} = 11{,}000$ mm^2

$A_{BC} = 3{,}900$ mm^2

(a) SHORTENING δ_{AC} OF THE TWO COLUMNS

$$\delta_{AC} = \sum \frac{N_i L_i}{E_i A_i} = \frac{N_{AB} L}{EA_{AB}} + \frac{N_{BC} L}{EA_{BC}}$$

$$= \frac{(1120 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(11{,}000 \text{ mm}^2)} + \frac{(400 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(3{,}900 \text{ mm}^2)}$$

$$= 1.8535 \text{ mm} + 1.8671 \text{ mm} = 3.7206 \text{ mm}$$

$\delta_{AC} = 3.72$ mm $\longleftarrow$

(b) ADDITIONAL LOAD P_0 AT POINT C

$(\delta_{AC})_{max} = 4.0$ mm

δ_0 = additional shortening of the two columns due to the load P_0

$$\delta_0 = (\delta_{AC})_{max} - \delta_{AC} = 4.0 \text{ mm} - 3.7206 \text{ mm} = 0.2794 \text{ mm}$$

$$\text{Also, } \delta_0 = \frac{P_0 L}{EA_{AB}} + \frac{P_0 L}{EA_{BC}} = \frac{P_0 L}{E}\left(\frac{1}{A_{AB}} + \frac{1}{A_{BC}}\right)$$

Solve for P_0:

$$P_0 = \frac{E\delta_0}{L}\left(\frac{A_{AB}A_{BC}}{A_{AB} + A_{BC}}\right)$$

SUBSTITUTE NUMERICAL VALUES:

$E = 206 \times 10^9$ N/m² $\quad \delta_0 = 0.2794 \times 10^{-3}$ m

$L = 3.75$ m $\quad A_{AB} = 11{,}000 \times 10^{-6}$ m²

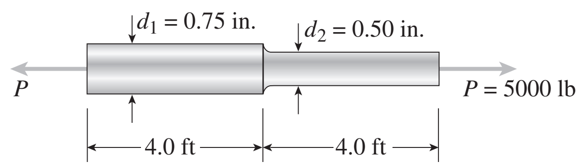
$A_{BC} = 3{,}900 \times 10^{-6}$ m²

$P_0 = 44{,}200$ N $= 44.2$ kN ⟵

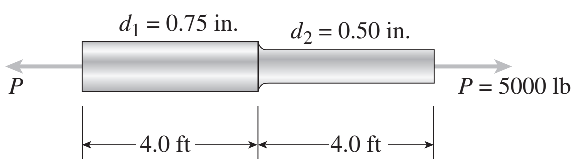
Problem 2.3-7 A steel bar 8.0 ft long has a circular cross section of diameter $d_1 = 0.75$ in. over one-half of its length and diameter $d_2 = 0.5$ in. over the other half (see figure). The modulus of elasticity $E = 30 \times 10^6$ psi.

(a) How much will the bar elongate under a tensile load $P = 5000$ lb?

(b) If the same volume of material is made into a bar of constant diameter d and length 8.0 ft, what will be the elongation under the same load P?



Solution 2.3-7 **Bar in tension**



$P = 5000$ lb

$E = 30 \times 10^6$ psi

$L = 4$ ft $= 48$ in.

(a) ELONGATION OF NONPRISMATIC BAR

$$\delta = \sum \frac{N_i L_i}{E_i A_i} = \frac{PL}{E}\sum\frac{1}{A_i}$$

$$\delta = \frac{(5000 \text{ lb})(48 \text{ in.})}{30 \times 10^6 \text{ psi}} \times \left[\frac{1}{\frac{\pi}{4}(0.75 \text{ in})^2} + \frac{1}{\frac{\pi}{4}(0.50 \text{ in.})^2}\right]$$

$= 0.0589$ in. ⟵

(B) ELONGATION OF PRISMATIC BAR OF SAME VOLUME

Original bar: $V_o = A_1L + A_2L = L(A_1 + A_2)$

Prismatic bar: $V_p = A_p(2L)$

Equate volumes and solve for A_p:

$V_o = V_p \qquad L(A_1 + A_2) = A_p(2L)$

$$A_p = \frac{A_1 + A_2}{2} = \frac{1}{2}\left(\frac{\pi}{4}\right)(d_1^2 + d_2^2)$$

$$= \frac{\pi}{8}[(0.75 \text{ in.})^2 + (0.50 \text{ in.})^2] = 0.3191 \text{ in.}^2$$

$$\delta = \frac{P(2L)}{EA_p} = \frac{(5000 \text{ lb})(2)(48 \text{ in.})}{(30 \times 10^6 \text{ psi})(0.3191 \text{ in.}^2)}$$

$= 0.0501$ in. ⟵

NOTE: A prismatic bar of the same volume will *always* have a smaller change in length than will a nonprismatic bar, provided the constant axial load P, modulus E, and total length L are the same.